dov Bootcamp 2020

peter.gruber@usi.ch

Course Description

This module lays out the foundations for the digitise-optimise-visualise summer school. Topics include: the formulation of an optimisation problem, a recap of relevant linear algebra, a mathematical toolbox (convex sets, convex functions and basics of convex analysis), a Python bootcamp and an introduction to visualising both mathematical phenomena and data.

Course Objectives

The course aims at opening the world of convex optimisation to the students by providing them with the mathematical and computational foundations. Three equally important approaches will be used to introduce the relevant mathematical concepts: (i) formally, (ii) visually and (iii) with applications in Python.

Prerequisites

In order to fully profit from the course, students should have some knowledge of:

- Basic mathematics as defined in
    - Matematica 1 [http://search.usi.ch/it/corsi/35259888/matematica-i]
    - Matematica 2 [http://search.usi.ch/it/corsi/35259987/matematica-ii]
- Basic experience with numerical computing
- Basic programming with Python – *or* – having completed the following datacamp introduction *before* the course starts (free access will be provided):
    - Introduction to Python
    - Intermediate Python for Data Science
    - Data science tool box 1

Course Work and Grading

The course is organised as a 1-week block course. Morning sessions focus on theory, followed by visualisations, applications, PC labs and exercises in the afternoon. Active class participation contributes 30% to the grade. An individual take-home exam (24 hours), combined with a short oral colloquium, contributes the remaining 70%.

Literature and Course Material

- Course slides
- Jupyter notebooks and sample code
- Cheat sheets
    - Notation
    - Python and CVXopt reference
    - Programming style
- Free access to web resources on datacamp.com
- Book: Boyd and Vandenberghe: Convex Optimization

Detailed Lecture plan

Day 1

Introduction -- start with why

- Why optimise? Why focus on convex optimization?
- Why use a formal / geometric / visual approach?
- Why use Python and CVXopt?

The canonical optimization problem

- Formal notation of an optimization problem with constraints
- Examples
- What we already know about optima ... (and partly should forget)

Normed linear vector spaces

- Vectors and spaces
- Linear independence
- Bases
- Norms and implied choices
- Banach and Hilbert spaces, convergence
- Classic inequalities
- Inner products and orthogonality
- Adjoints
- Projections
- Linear functionals
- Linear operators

Exercises in linear algebra

- From linear equation to Matrix notation
- Matrix properties and matrix calculus

PC lab: Getting started with computation

- The nuvolus platform and Jupyter notebooks
- A quick recap of Python: functions and linear algebra

Visualisation lab

- Visualising vectors, planes, half-spaces
- Projections
- Matrices

Day 2

Convexity and cones

- Convex and affine sets
- Cones
- Convex hull
- Convex cones

More linear vector spaces

- Dual spaces, dual norms
- Some new concepts: halfspaces, hyperplanes, polyhedra

More linear algebra

- Spectral analysis
- Matrix decompositions

Paper lab: recognise and verify convex sets

- Visual exercises
- Analytical exercises

Visualisation lab

- Visualising convex sets, convex cones, norm balls

Day 3

Convexity and duality

- Dual cones, norm cones
- Operations that preserve convexity in sets
- Generalised inequalities and applications
- Hyperplane theorems
- Convex functions: definition, properties

Paper lab: convex calculus

- (De)construct convex sets
- Visualise convex sets by hand

Exercises

- Recognise and verify convex functions

Working with data

- Random variables and measurement
- A few relevant datasets and sources

PC lab: Data

- Accessing, merging, subsetting data in Python
- Organising data

PC lab: Visualisation

- Visualising the concept of convexity
- Visualising (convex) data sets

— Weekend break —

Day 4

Differential calculus

- Differential calculus in higher dimensions
- Advanced differentiability criteria
- Matrix derivatives

Visualisation theory

- Perception
- Principles of design
- Types of visualisations

Exercises

- Applications of derivatives: quadratic forms, OLS

PC lab: Functions

- Floating point representation
- Numeric derivatives
- Implement and visualise functions in Python
- Parametric curves
- Verify convexity on projections
- Extended-value extension and ODE (out of domain errors)

Visualisation lab

- Functions
- Derivatives in higher dimensions

Day 5

The optimization problem

- Notation, classification, constraints

Visualisation lab: Visual optimization

- Visualising objective functions
- Visualising linear and cone-based constraints
- Visual solutions to optimization problems

PClab: CVXopt library

- Interface
- Formulation of problems and constraints
- Interpretation of outputs

Optimization algorithms

- Convergence
- How CVXopt works in principle
- Dangers and pitfalls in numerical algorithms